

# On the use of MPT to derive optimal RES electricity generation mixes

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## Abstract:

The use of modern portfolio theory (MPT) is a common practice to derive efficient frontiers and support portfolio decision making in financial markets. Although real projects present different characteristics and technical restrictions, the general objective of the decision maker is the same: to maximize the expected return minimizing the portfolio risk. Long term electricity generation decision making is characterized by high uncertainty, high impact on social welfare and a large set of diversified technologies that may be included in future scenarios. The possibility of applying MPT approach to define efficient electricity generation portfolios is explored in this paper focusing on particular in renewable energy sources (RES technologies). The use of MPT for building RES scenarios is demonstrated for the particular case of Portugal. One year hourly data concerning power output from wind, hydro and solar plants along with the power demand was collected and included in the analysis. Three different approaches were considered for designing the efficient frontiers aiming at maximizing the RES electricity generation, minimizing deviation between the demand and the RES production and minimizing the levelised cost of the RES system. The results demonstrate how this approach can be an effective tool to support decision making but put also in evidence the need to build modified MPT models in order to take into account the technical restrictions of the system.

## Keywords:

Renewable electricity sources, Electricity generation, Modern portfolio theory.

## 1. Introduction

Electricity power planning relates to generation, transmission and distribution systems. In this paper we are focused only on the generation system.

The main goal of generation planning is to meet customers' electricity needs at least cost with an acceptable degree of safety, reliability and quality [1]. However, this is a difficult task given that generation planning deals with future decisions that have to be made in an environment of uncertainty (namely, due to electricity demand, fuel prices volatility, investment costs, regulatory framework) and such uncertainties have to be, explicitly, taken into account in electricity planning [2]. In order to achieve this goal, it is necessary to couple supply-side management programs (which involve the construction of new power plants and/or repowering of existing ones) with demand-side management programs (in order to manage the customer load demand) [3].

In short, generation planning tasks include energy and demand forecasting, supply-side management and demand-side management adjustments, analysis of alternative expansion plans, determination of the optimal strategy or portfolio strategies and the evaluation of financial implications and feasibility [1].

Traditionally, the least-cost approach has been used in generation planning. This approach is frequently based on calculating the levelised costs of electricity generation, expressed in €/MWh, for different alternative technologies (e.g. fossil fuels, nuclear, renewable) and comparing such costs in order to choose the technology with the lowest cost.

However, some criticisms to the use of this approach can be found in the literature. Firstly, the fact that electricity planning decision makers are faced both with a wider range of alternative technologies for electricity generation and different institutional framework in which they operate, coupled with a future that appears increasingly complex and uncertain [4].

Secondly, as energy markets have been liberalised, the interest in quantifying and manage market risks grew [5]. In fact, with the deregulation and liberalisation of electricity markets, with a corresponding increase in competition, electricity generation companies will no longer have a guaranteed return because the price of electricity varies depending on a number of factors. In this context, it is essential that those companies can manage electricity price risk [6].

Additionally, there is the issue of security of energy supply [7]. In fact, given the global shortage in terms of primary fuel sources, policy makers increasingly need to consider a diversification of electricity production. Simultaneously, the price volatility of fossil fuels raises the question of what are the best options in terms of energy needs of a country.

Finally, an important feature of renewable technologies is that they correspond to capital intensive investments, which translates into a relatively fixed cost structure over time, with very low (or practically zero) marginal costs, and that are uncorrelated with important risk drivers, such as fossil fuel prices [6,7].

Given these reasons, it is necessary to shift from a paradigm that seeks to evaluate different technologies for electricity production on a stand-alone basis, to one that evaluate different portfolios of technologies for electricity production [4,7]. This means abandoning the traditional least-cost approach and to adopt a new perspective of analysis based on the theory of efficient portfolios. In this context, the "mean-variance portfolio (MVP) theory is highly suited to the problem of planning and evaluating a nation's electricity portfolio and strategies" [4]. Although, "at any given time, some alternatives in the portfolio may have higher costs while others have lower costs, yet over time, the astute combination of resources serves to minimize overall expected generating cost relative to the expected risk" [4].

In the context of electricity planning, where a combination of conventional technologies and renewable technologies is being considered, although renewables may present a higher levelised cost, it does not necessarily mean that the overall cost of the portfolio of technologies become more expensive, given the "statistical independence of renewables costs, which do not correlate (or covary) with fossil price movements" [4]. In fact, the inclusion of renewable technologies in an electricity generation portfolio is a way to reduce the cost and risk of the portfolio, although in a stand-alone basis the cost of those renewable technologies might be higher [7].

The electricity generation sector is essential for the attainment of the European renewable objectives. According to the European Union (EU) forecasts, the large hydropower will maintain its dominant position in renewable energy sources (RES) for electricity generation for the near future. However, the use of wind will continue expanding and, in 2020, the onshore and offshore wind electricity generation will overcome the hydro sector in the EU-27. Biomass/waste remains as the third RES for electricity (RES-E) technology with two digit RES share. An increase of the solar technologies is also foreseen although staying far from the wind, hydro or biomass shares [8].

The definition of optimal scenarios for RES-E to include on the grid has been frequently debated in the literature adopting multicriteria tools or electricity planning models based on cost/emissions optimization procedures. However, more recently the importance of diverse electricity technologies portfolios has been also emphasised and the use of the modern portfolio theory (MPT), previously established for the financial investment analysis, has been well applied to the electricity generation sector. This paper applies MPT as an electricity generation planning tool, in order to present optimal RES electricity generation mixes for the future, taking into account the past production pattern of each RES and optimizing the trade-off between maximizing RES output and minimizing RES variability.

The rest of the paper is organised as follows. In section 2, a brief description of the MPT reasoning and its application to electricity planning is presented. Section 3 corresponds to the empirical study undertaken, regarding the optimal RES electricity portfolios in Portugal. Finally, Section 4 draws the main conclusions of this paper and presents perspectives for future research.

## 2. Modern Portfolio Theory for energy decisions

### 2.1. Brief overview of MPT theory

Modern portfolio theory has its roots in the seminal paper by [9]. He proposed a methodology to select efficient investment portfolios based on investors' goal of maximising future expected return given a certain level of risk they were willing to take [10].

Investors in financial assets expect to earn a certain return over a given investment horizon. However, the yield actually obtained by the investor may differ from the expected return, and this represents the investment's source of risk. When deciding about his investments, the investor should consider, besides expected return, the following elements [11]: the dispersion of returns around the average return (variance), the symmetry of the distribution (skewness), and the kurtosis of the distribution. However, one of the innovations of the mean-variance model of [9], was the assumption that the distribution of returns follows a normal distribution. This has the advantage of being able to ignore those last two elements because the normal distribution is symmetric and has a kurtosis of zero. Thus, the characteristics of these investments can be measured based on only two variables: expected return and variance [11]. Therefore, assuming the assumption that investors are risk averse, having to choose between two investments with the same standard deviation but different expected returns, they always choose the one with higher expected return (and vice versa).

Thus, the mean-variance model allowed to explain the advantages that an investor has to diversify their investments among several securities (e.g. stocks or bonds). That is, instead of investing in a single asset, investing in portfolios made up of various financial assets. In fact, there are two reasons why diversification reduces the risk of investment [11]. On the one hand, as each asset included in a diversified portfolio represents a small portion of the investor's total investment, any event affecting one or a few of these assets have a more limited impact on the total value of the portfolio. On the other hand, the effect of specific events on the price of each asset included in a portfolio can be positive or negative. In large and well diversified portfolios, these effects tend to offset each other without affecting significantly the overall value of the portfolio.

One can illustrate the effects of diversification on the risk of a portfolio by examining the effect of adding more assets to the portfolio and see what happens to its variance. For example, in the case of a portfolio, P, consisting of two assets, A and B, expected return,  $E(r_p)$ , and variance,  $\sigma_p^2$ , are given by, respectively:

$$E(r_p) = \omega_A E(r_A) + \omega_B E(r_B) \quad (1)$$

and

$$\sigma_p^2 = 2\omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + 2\omega_A \omega_B \rho_{AB} \sigma_A \sigma_B \quad (2)$$

where  $\omega_A$  and  $\omega_B$  represent the proportions invested in each asset, A and B. The last term in the expression of the variance is often written in terms of the covariance of returns between two assets:  $\sigma_{AB} = \rho_{AB} \sigma_A \sigma_B$ . One can see that the benefits of diversification are a function of the correlation coefficient. Thus, the lower the correlation of returns between two assets the higher the gains from diversification an investor obtain.

This reasoning can be generalised for the case of a portfolio with N assets. Thus, expected return,  $E(r_p)$ , and variance,  $\sigma_p^2$ , of the portfolio are given by:

$$E(r_p) = \sum_{i=1}^{i=N} \omega_i E(r_i)$$

(3)

and

$$\sigma_p^2 = \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \omega_i \omega_j \rho_{ij} \sigma_i \sigma_j \quad (4)$$

We conclude, therefore, that the variance of a portfolio is partially determined by the variance of individual assets and partly by the way they move together. The latter is measured statistically by the coefficient of correlation or the covariance of the assets belonging to the portfolio. It is the term for the covariance that provides an explanation of why and in what amount diversification reduces the risk of investment. In fact, portfolios of financial assets should not be chosen only by their individual characteristics, but taking into account how the correlation between assets affects the overall risk of a portfolio [11]. Therefore, since the variances can be estimated for portfolios consisting of a large number of assets, suggests an approach to the optimal selection of portfolios in which investors make the balance between expected return and risk.

Alternative 1: If an investor can specify the maximum risk he is willing to take, the optimal portfolio is obtained maximising expected return subject to that risk level, i.e.:

Alternative 2: If an investor specifies his desired level of expected return, the optimal portfolio is the one that minimizes the variance subject to that level of return:

$$\text{Max} E(r_p) = \sum_{i=1}^{i=N} \omega_i E(r_i)$$

s.t.

$$\sigma_p^2 = \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \omega_i \omega_j \sigma_{ij} \leq \hat{\sigma}^2$$

$$\sum_{i=1}^N \omega_i = 1$$

$$\omega_i \geq 0$$

$$\text{Min} \sigma_p^2 = \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \omega_i \omega_j \sigma_{ij}$$

s.a.

$$E(r_p) = \sum_{i=1}^{i=N} \omega_i E(r_i) = E(\bar{r})$$

$$\sum_{i=1}^N \omega_i = 1$$

$$\omega_i \geq 0$$

The portfolios that result from this process give rise to what is called the efficient frontier, as represented in Figure 1:

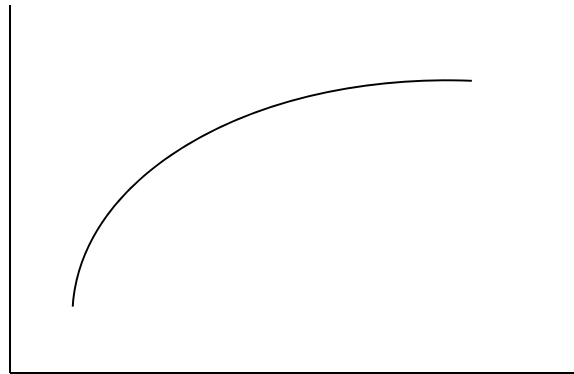


Fig. 1. – Efficient frontier

## 2.2. MPT applications to electricity generation

In recent years there has been a growing application of the MPT theory to electricity planning. In fact, the mean-variance model can be used to determine the optimal portfolios of electricity generation both for a company or a country. According to [5], the main idea of the MPT model is that the value of each asset can only be determined taken into account portfolios of alternative assets. Hence, energy planning should be focused more on developing efficient production portfolios and less on finding the alternative with the lowest production cost [4,7].

The MPT approach allows to analyse the impact of the inclusion of renewable technologies in the mix of generating sources of electricity. In particular, it provides a better risk assessment of alternative generation technologies, something that the traditional stand-alone least cost approach cannot do, particularly in terms of the impact of renewable energy sources in reducing the risk of the portfolio of technologies to be adopted. In fact, the MPT model allows to illustrate the trade-off between production costs and risk: the lower the cost the higher the risk, meaning that it is not possible to achieve a lower electricity production cost without assuming higher levels of risk.

It should be noted that the result of applying the mean-variance model to generation planning is not identifying a specific portfolio, but the identification of an efficient frontier where the optimal portfolios will be located. These are Pareto-optimal, that is, an increase in returns (or a decrease in costs) is only achieved by accepting an increased risk. On the other hand, an important aspect in the mean-variance model is the assumption that past events are the best guide for predicting the future. Not to say that unexpected events will not occur, but that the effect of these events is already known from past experience [7].

A study that used the MPT theory to obtain evidence about the best mix of electricity generation in Scotland was that of [12]. Based on the efficient frontier, the authors analysed the portfolios suggested in four scenarios for the electricity generation mix in 2020, seeking to clarify what role renewable technologies can play in setting up those portfolios. The main conclusions reached by those authors were that: the portfolios of electricity production corresponding to the four scenarios are not mean-variance efficient; based on MPT approach it is possible to quantify the likely scale of inefficiency; and it seems there is the opportunity to have an improvement in the generation mix in the sense of Pareto.

Another study was conducted by [13], where they tried to optimise wind power investment portfolios across countries taking into account the correlation between wind farms output located in different geographical areas. In fact, the aim is "to demonstrate the use of MVP theory as an insightful analytical approach to take into account the impact of wind output variability and correlations of wind output across different locations within a wind farm portfolio" [13]. These authors concluded that the current and projected portfolios for 2020 are far from the efficient frontier and, therefore, there is scope for wider benefits arising from greater coordination of European renewable development by providing "incentives for location of new wind farms so as to maximise the efficiency of the overall European wind portfolio".

In turn, [5] apply the MPT theory in order to optimise generation electricity portfolios but focusing their attention "on private investors' investment incentives in liberalized electricity markets, where fuel-mix diversification is a possible strategy for reducing exposure to electricity, fuel, and carbon price risks". In fact, according to these authors, the electric utilities operating in deregulated markets cannot easily pass on to the sales price changes in their production costs. Thus, utilities have to take into account the risks that may affect their profits when they have to decide about its investment projects. In this context, the risks regarding electricity, fuel and carbon prices become relevant in determining the optimal production portfolios. The results obtained by [5] demonstrated the importance of the degree of correlation between the prices of electricity, fuel and carbon in the definition of the optimal generation mix. Hence, they concluded that "liberalized electricity markets characterized by strong correlation between electricity and gas prices [...] are unlikely to reward

fuel mix diversification sufficiently to make private investors' choices align with the socially optimal fuel-mix, unless investors can find counterparties with complementary risk profiles to sign long-term power purchase agreements".

Also from the perspective of a private generation company, operating in a liberalised electricity market, [6] applied the theory of efficient portfolios. In this type of markets, it is essential that utilities companies can properly manage the electricity price risk, given the strong competition among the different operators in those markets. To address this issue, [6] adopt the MPT approach in order to define the best strategy for electricity trading for a company that is considering selling in the spot market or establish bilateral contracts. The question that arises is "how to allocate energy among these potential transactions in order to maximize profits with relatively low risk" [6]. In fact, the combination of different trading strategies of electricity can be seen as constituting a portfolio which can be optimised using the MPT approach.

Finally, [4] presents a summary of the application of MPT theory in the evaluation of different electricity generation planning scenarios for the case of U.S., EU and Mexico, where was perceived that the mix of electricity generation can be improved in terms of cost and/or risk, by expanding the use of renewable technologies. The author states that "compared to existing, fossil-dominated mixes, efficient portfolios reduce generating cost while including greater renewables shares in the mix thereby enhancing energy security. Though counterintuitive, the idea that adding more costly renewables can actually reduce portfolio-generating cost is consistent with basic finance theory". It follows an important conclusion: "in dynamic and uncertain environments, the relative value of generating technologies must be determined not by evaluating alternative resources, but by evaluating alternative resource portfolios" [4].

The above mentioned papers demonstrate the possibility of adapting a financial theory on electricity planning problems. In fact, the increase of RES to electricity generation creates important challenges to grid managers due to the expected variability of the power output of most of these RES power plants. The adoption of a model based on MPT can be particularly useful for electricity systems highly RES supported, allowing to take into account both yearly seasonality and intra-daily variations of the production. This paper proposes to demonstrate the use of MPT on these systems resourcing to the particular case of the Portuguese electricity system to identify optimal RES portfolios. The aim is to optimize the trade-off between the variable production that characterize some of the RES and the return of these projects, measured according to a set of proxy variables.

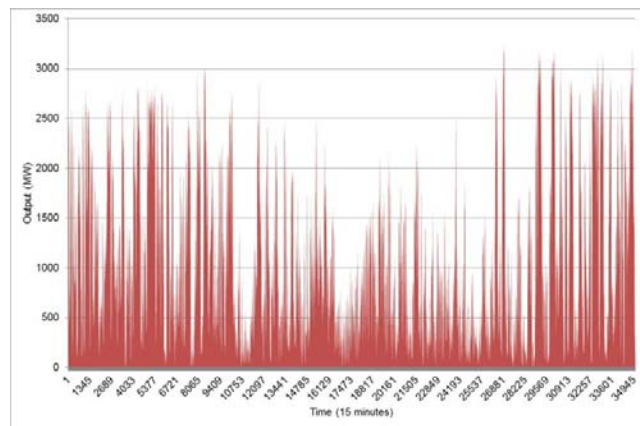
### **3. Optimal RES electricity portfolios**

The Portuguese electricity system is mainly based on a mix of thermal, hydro and wind power technologies. RES power plants represent 54% of the total installed power. The wind sector grew rapidly in the last years and an increase on the hydropower investment is also foreseen for the next years, strongly justified by the need to compensate the variable output of wind power plants. As in the EU-27, biomass represents an important RES contributor, mainly because of industrial wastes used in CHP and, in much smaller amount, by the centralized biomass power plants [14].

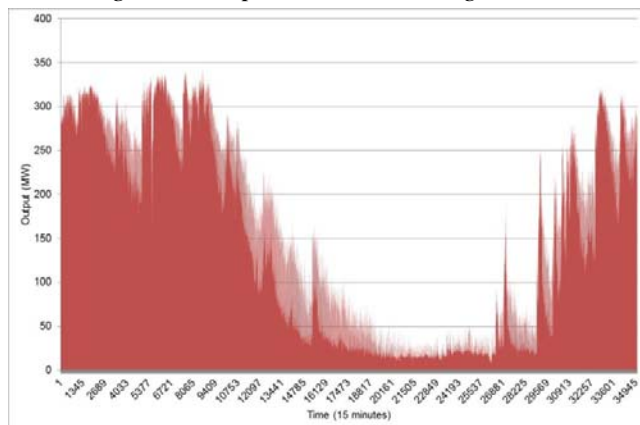
Some recent studies already addressed the case of electricity generation scenarios in Portugal and the use of optimization models to draw these scenarios [15,16]. However, to the authors' best knowledge no attempt has been made to use an approach close to the MPT theory to this system. In fact, most optimization models rely on the cost and/or emissions minimization of the electricity system. Functions such as the loss of load probability or the reserve margin are used to address the minimum requirements for security of supply. These functions although allowing to include the variability of RES power output do not explicitly recognize portfolio risk as a decision variable influenced by the risk of each technology output and, most importantly, by the correlations between those risks. The general idea of this research is to present possible RES generation mixes that would

ensure maximum return (or minimum cost) for each given portfolio risk level, obtaining then the efficient frontier. The use of the Portuguese case, as an electricity system strongly influenced by RES seasonality behaviour, is expected to contribute to demonstrate how MPT can provide a way to complement cost optimization models with a quantitative risk evaluation of the electricity generation portfolio.

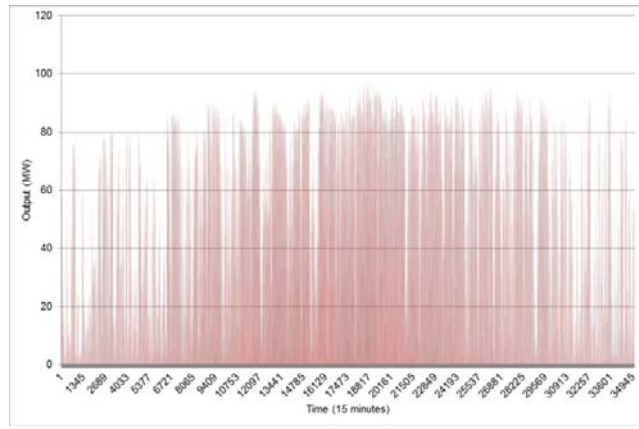
The data used for the models was drawn from public information available on REN site ([www.ren.pt](http://www.ren.pt)), consisting of the load output of each RES power plant measured for each quarter of an hour for an one year period. For the case presented in this paper, 2010 information was considered representing 35040 measures for each technology. This allowed to capture the daily and yearly seasonality of RES technologies output and of the demand. Figures 2 to 5 show the load output of wind, small hydro, photovoltaic and small thermal power plants (including renewable and non-renewable cogeneration and biomass power plants).



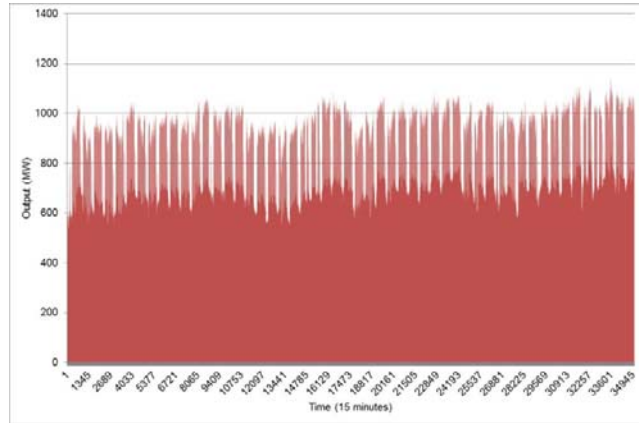
*Fig. 2. Wind power load, Portugal 2010.*



*Fig. 3. Small hydro load, Portugal, 2010*



*Fig. 4. Photovoltaic load, Portugal, 2010.*



*Fig. 5. Small thermal power load, Portugal, 2010*

From the figures it became evident that the variability of the RES output comes mainly from the non-storage RES production, namely wind, hydro and photovoltaic power plants. The Portuguese system includes also large dams and run of river hydro plants, each one of them with some storage capacity. Although storage capacity of run of river power plants is limited, it also allows reducing the variability of the hydro power output. As for the small hydro power plants most of them do not present storage capacity and as so it was assumed that their production could represent a proxy variable for the hydro availability. Both the wind power and photovoltaic loads were assumed as proxy variables for the underlying resource availability. Being possible to storage, the variability of the biomass power output is much lower than the all the other RES and does not depend on the hourly availability of the resource. For this reason, only, wind, hydro and sun technologies are included in this analysis.

To make the variables comparable, the output of each technology was normalized by the installed power in 2010, as described in (5).

$$L_{i,t} = \frac{\text{Output}_{i,t}}{\text{Installed power}_i} \quad (5)$$

Where  $i$  represents the technology (1- wind; 2- hydro; 3- photovoltaic),  $t$  represents the moment in time and  $L_{i,t}$  represents the normalized variable for each technology in each quarter of an hour.

The demand was also used on the second model proposed, aiming to find the best RES solution that could meet the desired demand with the lowest deviation. For this an additional proxy variable was used to normalise the demand by the peak load, as described in (6).

$$LD_t = \frac{\text{Demand}_t}{\text{Peak load}} \quad (6)$$



Where  $LD_{i,t}$  represents the normalized demand in each quarter of an hour.

The proxy variables included on the proposed MPT models are characterized in Table 1 and include:

- Normalized wind power output, representing the wind availability of the system.
- Normalized small hydro output, representing the hydro inflows (hydro availability) to the system.
- Normalized photovoltaic output, representing the sun availability of the system.
- Normalized demand, representing the electricity needs of the system

*Table 1. Characteristics of the proxy variables for MPT model.*

	Wind	Hydro	Photovoltaic	Demand
Mean (MW/Installed MW)	0,278	0,383	0,194	0,634
Standard deviation (MW/Installed MW)	0,210	0,281	0,264	0,120
Correlation coefficient				
<i>Wind</i>	1	0,335	-0,255	0,0019
<i>Hydro</i>		1	-0,152	0,0105
<i>Photovoltaic</i>			1	0,0080
<i>Demand</i>				1

In the following sections different scenarios will be presented applying models based on the MPT theory. Three different approaches were considered for designing the efficient frontiers: (1) maximizing the RES-E generation (MPT\_RES); (2) minimizing the difference between demand and RES-E production (MPT\_RES@Demand); (3) minimizing RES cost scenarios, according to the expected levelized cost of each technology (MPT\_RES@Cost). Optimization models were built and Excel Solver was used to find optimal solutions for each problem.

### 3.1. MPT\_RES model

For this analysis a traditional MPT model was used aiming to design the efficient frontier that can maximize the expected RES production per unit of installed capacity for each risk level. The optimisation model is described by (7) to (10).

Objective function

$$\text{Max } E(L_p) = \sum_{i=1}^3 W_i E(L_i) \quad (7)$$

Restrictions

$$\sigma(L_p) = \sqrt{\sum_{i=1}^3 W_i^2 \sigma_i^2 + \sum_{i=1}^3 \sum_{k=1(k \neq i)}^3 W_i W_k \rho_{ik} \sigma_i \sigma_k} \quad (8)$$

$$\sum_{i=1}^3 W_i = 1 \quad (9)$$

$$W_i \geq 0 \quad \forall_i \quad (10)$$

Where  $E(L_p)$  represents expected return of the portfolio (RES generation per installed MW),  $W_i$  represents the share of technology  $i$ ,  $E(L_i)$  represents the expected  $i$  technology output ( $i$  generation per installed MW),  $\sigma(L_p)$  represents the standard deviation of the portfolio,  $\sigma_i$  represents the standard deviation of  $i$  technology output, and  $\rho_{ik}$  represents the correlation coefficient between  $i$  and  $k$  technologies outputs.

Figure 6 and Table 2 describe the results obtained, including the efficient frontier and the characterization of a set of optimal portfolios. Figure 6 presents also the present RES (wind, hydro and photovoltaic) portfolio and the expected one in 2022, according to REN forecast [15,16].

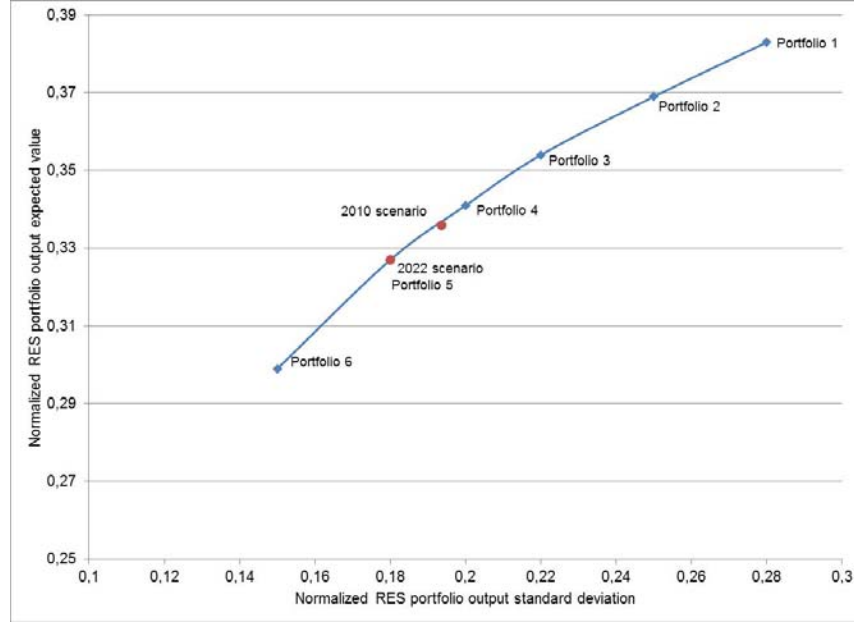


Fig. 6. Efficient frontier MPT\_RES model

Table 2. Characterization of MPT\_RES optimal portfolios

	$\sigma(L_p)$	$E(L_p)$	Wind	Hydro	Photovoltaic
Portfolio 1	0,28	0,383	0,30%	99,70%	0%
Portfolio 2	0,25	0,369	13,00%	87,00%	0%
Portfolio 3	0,22	0,354	27,82%	72,18%	0%
Portfolio 4	0,2	0,341	34,58%	62,50%	2,92%
Portfolio 5	0,18	0,327	36,54%	54,13%	9,33%
Portfolio 6	0,15	0,299	40,68%	37,52%	21,80%
2010 Scenario	0,194	0,336	42,03%	56,59%	1,38%
2022 Scenario	0,18	0,327	38,22%	53,46%	8,32%

### 3.2. MPT\_RES@Demand model

For this analysis a modified MPT model was used aiming to design the efficient frontier that can minimise the deviation between the demand and the RES production in each moment. The idea is to define optimal RES portfolios that can contribute to better meet the demand in each moment, following a close load distribution pattern. The proposed optimisation model is described by (11) to (14).

Objective function

$$\text{Min } E(L_p) = E(LD) - \sum_{i=1}^3 W_i E(L_i) \quad (11)$$

Restrictions

$$\sigma(L_p) = \sqrt{\sum_{i=1}^3 W_i^2 \sigma_i^2 + \sigma_d^2 + \sum_{i=1}^3 \sum_{k=1(k \neq i)}^3 W_i W_k \rho_{ik} \sigma_i \sigma_k - \sum_{i=1}^3 W_i \rho_{id} \sigma_i \sigma_d} \quad (12)$$

$$\sum_{i=1}^3 W_i = 1 \quad (13)$$

$$W_i \geq 0 \quad \forall_i \quad (14)$$

Where,  $\sigma_d$  represents the standard deviation of the demand and  $\rho_{id}$  represents the correlation coefficient between  $i$  k technologies outputs and the demand.

From the reduction of risk perspective, a negative correlation between technologies is desirable to ensure their complementarity. However, but a positive correlation between RES technologies output and the demand should lead also to risk reduction under this model. The traditional standard deviation calculation was changed taking this into consideration, as may be seen in (12).

Figure 7 and Table 3 describe the results obtained, including the efficient frontier and the characterization of a set of optimal portfolios.

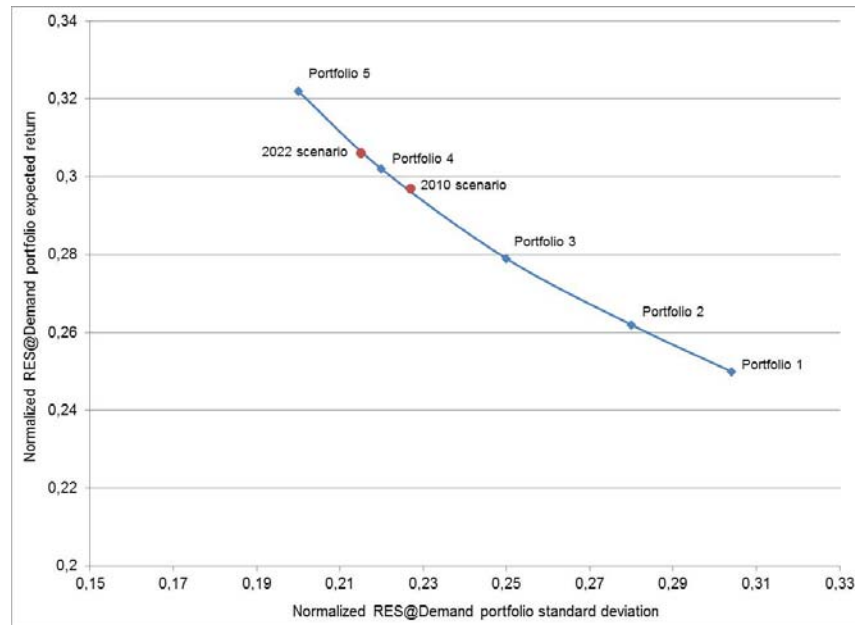


Fig. 7. Efficient frontier MPT\_RES@Demand model

Table 3. Characterization of MPT\_RES@Demand optimal portfolios

	$\sigma(L_p)$	$E(L_p)$	Wind	Hydro	Photovoltaic
Portfolio 1	0,304	0,25	0%	100%	0%
Portfolio 2	0,28	0,262	11,48%	88,52%	0%
Portfolio 3	0,25	0,279	27,87%	72,13%	0%
Portfolio 4	0,22	0,302	35,87%	56,40%	7,73%
Portfolio 5	0,20	0,322	38,76%	44,52%	16,72%
201 Scenario	0,227	0,297	42,03%	56,59%	1,38%
2022 Scenario	0,215	0,306	38,22%	53,46%	8,32%

### 3.3. MPT\_RES@Cost model

This analysis is similar to the one conducted in section 3.1. However, the model is now weighted by the levelised costs of each RES technology. This way, an efficient frontier will be drawn from the optimization model with the objective goal being the minimization of the total expected cost of the RES system. The optimization model is described by (15) to (18) describe the model.

Objective function

$$\text{Min } E(LC_p) = \sum_{i=1}^3 W_i LC_i E(L_i) \quad (15)$$

Restrictions

$$\sigma(LC_p) = \sqrt{\sum_{i=1}^3 W_i^2 LC_i^2 \sigma_i^2 + \sum_{i=1}^3 \sum_{k=1(k \neq i)}^3 W_i W_k \rho_{ik} \sigma_i LC_i \sigma_k LC_k} \quad (16)$$

$$\sum_{i=1}^3 W_i = 1 \quad (17)$$

$$W_i \geq 0 \quad \forall_i \quad (18)$$

Where  $E(LC_p)$  represents the expected levelised cost of the portfolio per unit of installed capacity,  $\sigma(LC_p)$  represents the standard deviation of levelised cost of the portfolio and  $LC_i$  represents the levelised cost of each  $i$  technology .

Figure 8 and Table 4 describe the results obtained, including the efficient frontier and the characterization of a set of optimal portfolios.

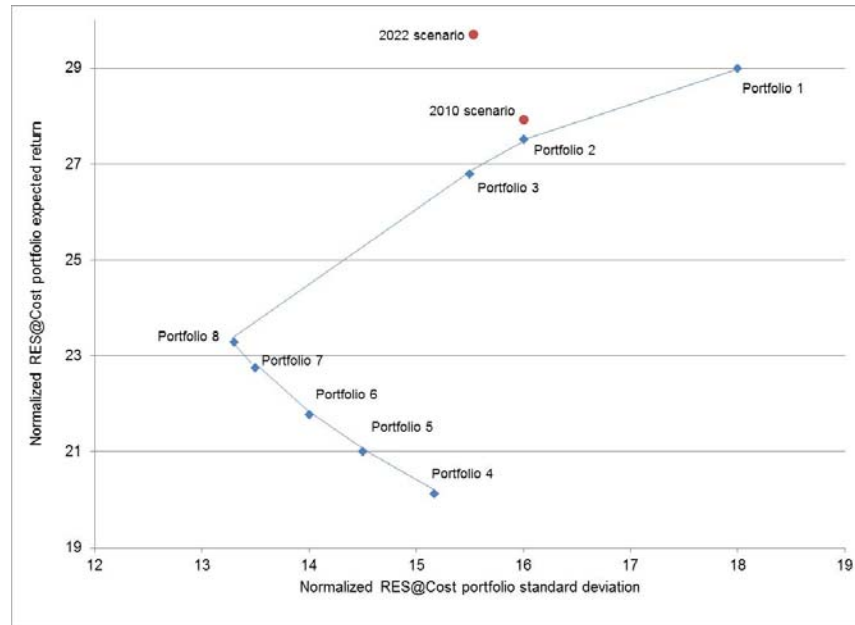


Fig. 8. Efficient frontier MPT\_RES@Cost model

Table 4. Characterization of MPT\_RES@Cost optimal portfolios

	$\sigma(LC_p)$	$E(LC_p)$	Wind	Hydro	Photovoltaic
Portfolio 1	18	29	31,80%	68,20%	0
Portfolio 2	16	27,51	44,84%	55,16%	0
Portfolio 3	15,5	26,8	48,80%	51,20%	0
Portfolio 4	15,17	20,13	100%	0	0
Portfolio 5	14,5	21	94,36%	5,64%	
Portfolio 6	14	21,77	89,90%	8,28%	1,82%
Portfolio 7	13,5	22,75	84,12%	12,74%	3,14%
Portfolio 8	13,3	23,29	81,03%	15,10%	3,87%
2010 Scenario	16	27,92	42,03%	56,59%	1,38%
2022 Scenario	15,53	29,71	38,22%	53,46%	8,32%

### 3.4. Analysis of results

The results indicate that both 2010 and 2022 scenarios [17,18] are close to the efficient frontier for MPT\_RES and MPT\_RES@Demand models. In fact, both these scenarios reflect the Portuguese energy policy goals of increasing RES share on the electricity system, diversifying the energy sources and promoting a strategy based on hydro reinforcement to deal with the increasing wind share. In the same way, most of the less risky scenarios described in figures 6 and 7 point to mix hydro-wind power scenarios as the more efficient ones. More risky strategies rely mainly on hydro power, the option with higher expected return but also the one with higher standard deviation. Although a positive correlation exists between wind and hydro, it does not seem to be enough to jeopardize the mix of these technologies in most of the scenarios. On the other hand, photovoltaic presents a less interesting expected value and a risk level close to the hydro one. It presents, however, the advantage of being negatively correlated to both wind and hydro. As so, less risky scenarios tend to include also this option combined with hydro and wind.

The MPT\_RES@Cost present quite different results, clearly driven by the levelised cost of the technologies. A strong reliance on wind power is evident along the efficient frontier, as this is the option with less expected cost and with the lowest standard deviation when considering the levelised cost normalized by the installed power. Solutions with lower risk are characterized by a mix of wind, hydro and to a much lower extent photovoltaic technology, leading to a higher expected cost but also taking advantage of the portfolio diversification.

Particularly interesting for the MPT\_RES@Cost is the comparison of portfolio 3, portfolio 4 and 2022 scenario. All of these solutions have close risk values, but very different expected levelised costs and RES structures. What seems to be the best solution (portfolio 4) is however, compromised by a 100% wind power share. From the technical point of view is a nonsense solution, due to the already existing hydro capacity and for motives of security of supply. Both portfolio 3 and 2022 scenario are much more balanced solutions, as a stronger diversity of the mix is foreseen.

The obtained results put in evidence the need to enrich the traditional MPT analysis with additional technical, legal and economic constraints when passing from financial markets to the analysis of portfolios of real projects. Traditional strategic electricity power planning cost optimization models with technical restrictions must be combined with efficient portfolio design with risk restrictions. The research project is now proceeding with this new approach into a single quantitative framework, envisaging the following elements:

- A cost objective: to minimize levelized cost of production of the electricity system as a whole.
- An environmental objective: to minimize environmental impacts, either measured by emissions or by externalities valuation.
- A risk objective: measured by the variance of the portfolio.
- A set of decision variables: share of each technology, measured by the ratio between the installed power of each technology and the total installed power of the system.
- A set of constraints: capacity limitations, legal and technical requirements and electricity demand needs.

## 4. Conclusion

Social welfare strongly depends on a reliable and competitive electricity system. RES technologies constitute key investments to design future scenarios or strategies for sustainable future. The raising trend of RES brings however considerable challenges to decision makers due to uncertainty of the production highly dependent on the availability of the underlying resources. This paper demonstrates the application of MPT for RES in electricity planning. This allowed to address both

the expected return and the RES portfolio risk, taking into account both the standard deviation of each technology output and the correlation coefficient between technology outputs and demand needs.

The study of the Portuguese case concludes that less risky solutions are characterised by a mix of RES technologies. This mix, however, depends on the criteria used to quantify the expected return. If the maximisation of the RES contribution to the system is the goal of the planner, hydro emerges as the major contributor. On the other hand, if decisions are driven by levelised costs, hydro is penalised and wind becomes the preferable option. Photovoltaic share only becomes relevant for low risky solutions, regardless of the model used. The present Portuguese RES generation mix and the forecasted scenario for 2022 [16, 17] showed to be close to the efficient frontier for the case of MPT\_RES and MPT\_RES@Demand models, reflecting the diversification goal for the sector. Notwithstanding, when the levelised cost is included in the analysis, both 2010 and 2022 scenarios move away from the efficient frontier.

Although the usefulness of the MPT approach in analysing the electrical planning scenarios, has been demonstrated, it is important not to forget some limitations of this approach. For example, [12] emphasised two issues. On the one hand, the failure to consider transaction costs associated with changes in generation mix. Second, the fact that, generally, the studies carried out do not take into account the feasibility of the efficient portfolios obtained with the MPT theory in the context of existing energy infrastructure. Moreover, [7] pointed out that the characteristics of electricity generation technologies are not always comparable to the characteristics of financial assets for which the MPT theory was developed. Firstly, markets for assets (e.g. turbines, coal plants) related to electricity generation are usually imperfect in contrast with capital markets, which also make them less liquid. Secondly, financial assets are almost infinitely divisible and fungible, which does not happen with electricity generating real assets. Finally, investments in electricity production technologies tend to be lumpy, especially renewable technologies. However, [7] consider that "for large service territories or for the analysis of national generating portfolios, the lumpiness of individual capacity additions becomes relatively less significant".

Recognizing that MPT for electricity system analysis must go beyond the traditional models, future work envisages the development of a new model combining MPT with generation expansion models for electricity power planning.

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